University of Saskatchewan
Department of Physics and Engineering Physics
EP 228.3

D.A. Degenstein

Midterm Examination, March 2004

ONE SMALL FORMULA SHEET ALLOWED

Time: 60 minutes

Instructions:

There are six questions each worth a different amount of marks
The exam is out of 49.
Read each question carefully and ***THINK*** before you ***ACT***.

1) Express each of the complex expressions in terms of $z = Me^{i\phi}$. Write down M and ϕ as single valued real numbers. Show some work or you get no marks. See last point in the instructions. ***(12 marks)***

i) $z = -2 + i3$

ii) $z = \dfrac{de^{-iz_1}}{dt}$ where $z_1 = \dfrac{\pi}{2}t - i0.3x$ evaluated at $t = 0$ and $x = 1$.

iii) $z = \left(9 + 4e^{-i\frac{\pi}{3}} - 4\left(\dfrac{1}{2} - i\dfrac{\sqrt{3}}{2}\right)\right)7e^{i\frac{\pi}{4}}\left(\dfrac{\sqrt{2}}{2} - i\dfrac{\sqrt{2}}{2}\right)$

2) What are the first three terms in the MacLaurin Series expansion of $e^{-x}\cos(2x)$? Remember the MacLaurin expansion is the Taylor Expansion about zero***(6 marks)***

3) Consider $z_1 = A_1e^{i\phi_1}$, $z_2 = A_2e^{i\phi_2}$ and $z_3 = A_3e^{i\phi_3}$ where $A_1 = 2$, $A_2 = \dfrac{1}{2}$, $A_3 = \dfrac{1}{4}$, $\phi_1 = \dfrac{2\pi}{3}t + \dfrac{\pi}{4}$, $\phi_2 = \dfrac{5\pi}{3}t$ and $\phi_3 = \dfrac{4\pi}{3}t$. What is the derivative of the function that describes the real part of $z_1 + z_2 + z_3$? ***(5 marks)***

4) Show, using the complex representation of $\cos\theta$ and $\sin\theta$, that: ***(8 marks)***

$$\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$$

5) Prove $1\times 2+2\times 3+3\times 4+\cdots+n(n+1)=\dfrac{n(n+1)(n+2)}{3}$ using the principles of induction and the basis case of $n=1$. *(8 marks)*

6) Given the set of data points :

$$(-2,12),(-1,7),(0,4),(1,3)\ and\ (2,4)$$

that were measured, without error, by sampling a phenomena that can be described by a perfect parabola, i.e. : $f(t)=a_2t^2+a_1t+a_0$

i) Determine the numerical integral of the sampled curve using 0^{th}, 1^{st} and 2^{nd} order polynomial approximations. **IMPORTANT**: There is a mistake to be made on this particular question that will result in the forfeiture of all *4* marks. *(4 marks)*

ii) Determine the central difference numerical derivative at the applicable points. *(2 marks)*

iii) What is the exact value of the integral? *(2 marks)*

iv) What are three equations required to determine the coefficients, a_2, a_1, and a_0, for the exact quadratic fit? DO NOT SOLVE JUST WRITE DOWN THE EQUATIONS!!! *(2 marks)*

End of Exam!

$$a_n = a_{(n-1)} + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

$$a_{(n+1)} = a_n + (n+1)(n+2) = \frac{(n+1)(n+2)(n+3)}{3}$$

$$\frac{(n+1)(n+2)(n+3)}{3} = a_{(n-1)} + n(n+1) + (n+1)(n+2)$$

$$\frac{(n+1)(n+2)(n+3)}{3} = \frac{n(n+1)(n+2)}{3} - n(n+1) + n(n+1) + (n+1)(n+2)$$

$$\frac{(n+1)(n+2)(n+3)}{3} = (n+1)(n+2) + \frac{n(n+1)(n+2)}{3}$$

$$(n+1)(n+2)(n+3) = 3(n+1)(n+2) + n(n+1)(n+2)$$

$$(n+1)(n+2)(n+3) = (3+n)(n+1)(n+2)$$